

**The Nitty-Gritty of Ordinary Level Equations**

**(With over 250 worked examples and questions from WAEC, NECO, NABTEB & JAMB)**

**By**

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**DEDICATION**

**This book is dedicated to my mentor, father and friend**

**Late Dr. Innocent Okwu Amechi (Dikeohamatics)**

**Founder of Dikeohamatics games and a great Mathematics pedagogist.**

**The news of his death came to me as a shock, mana anyi agaghị eje ọgụ be chukwu.**

**ACKNOWLEDGEMENTS**

**I wish to acknowledge My father, Chief Ede Ogbonna S. (Owelle), My Big Bro., Mr. Gabriel Edeh (Odogwu). The family of Mr. & Mrs. Onoh Paul Chinonso. The Family of Mrs. Roseline Anya, Professor Chinedu Aguba (Chairman ASUU, ESUT Branch). God’s blessings, I pray.**

**Special thanks to my mentors: Mr. Steven Etebefia. My very good friend and dad, Late Dr. Dikeoha Okwu Amaechi (Dikeohamatics), Mr. Ivans Ormsbee, Mr. Israel Oyibo, Mr. Ken Behrens (Proofreader of this book), Late Mr. Ikechukwu Samson Eze, Mr. Benyeogo Chinonso (Mr. Parker), Mrs. Akuma Felicia (School Mum), Mr. Valentine Ugorji, Mrs. Aniehe Angela, Mrs. Edet Joy. More especially to my Boss, Mr. Raymond Adie, proprietor of Raymond College Enugu & CEO R- Square Relaxation point. The man that gets facts easily from figures. The man that made me love calculations, gave me the necessary exposure & taught me how to relate Mathematics with things around me.**

**To you all, all your teachings and instructions aren’t insignificant.**

**I won’t fail to appreciate my friends that came to my rescue when I needed them most, with formidable questions. Friends like: Uzoma Victor, Precious Love Timilehin, Quadri Adeshina, Kingsley Ekamba, you are all blessed.**

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**Special Thanks to Sir Festus Kings, CEO “MY DREAMS ACADEMY”. God bless you now and always.**

**To my co-workers at Raymond College Abakpa Nike Enugu, Faith International School Ughelli Delta State, My Dreams Academy Enugu.**

**So much indebted to several authors without whose references, the authoring of this book will not have been a reality.**

**My Sincere gratitude to the CEO of JexGrey Global, Mr. Tobi Justin. You are held in high regard.**

**Foreword**

*This book has been simplified to the most basic level, and the examples arranged in proper ascending order. It has not only covered a lot of topics but treated them to the highest satisfactory level. The content is suitable for senior secondary school level, as well as students preparing for pre-degree and preliminary courses in universities, polytechnics and colleges of education. The exercises are numerous and widespread. Covering examinations of many years.*

*The author is an injector and generator. I commend the author Edeh Anthony Blaise Chidera for writing this book. It is a masterpiece. Those who would buy and study this book to the fullest, can be sure of performing very well in the selected topics, in any mathematics O’Level examination.*

*A tree cannot make a forest. The best of this book will include your constructive suggestions sent to the author.*

**Israel Chinedu Oyibo**

**(NCE Mth. /Phy. & BSc. Ed Math Edu.)**

**Foreword II**

*Many books have been written on General Secondary School Mathematics; some have been titled in a broader sense. It must be admitted that none I know is dedicated to the teaching of mathematical equations. The author of this book; the Nitty-Gritty of O’Level Equations has brought a new dimension to the stock of mathematical equations available. It is simply innovation.*

*The book presents to students the detailed approach of dealing with equations in mathematics. Teachers will find it very useful in the teaching of some of the topics covered. It served as a resource to colleagues in engaging in mathematical thinking to enrich and extend their own knowledge.*

*The book consists of fundamental topics necessary for clear understanding of solving mathematical equations and their concept*

*Linear Equations (One variable/ 2 variables), Simultaneous-Linear Equations. Quadratic Equations, Simultaneous-Quadratic Equations, Exponential (Indicial) Equations, Logarithm Equations, Radical Equations*

*Fractional Algebraic Equations, Trigonometric Equations, Number Base Equations, Factorial Equations, Modular Equations, Polynomial Equations, Matrix Equations etc.*

*Throughout the text insightful examples with acceptable analytical approach are given and at the end of several sections the reader will find numerous exercises arranged in order of difficulty which are designed to fill the text materials, follow the argument and provide a better understanding of the subject.*

*The exercises are intended to provide practices when dealing with the various special cases that can arise.*

*The reader may choose not to read this interesting book cover to cover as it’s a book that one may dip in and out of according to the specific needs.*

*However, I will advise the reader to embark on a comprehensive tour of the book.*

***Etebefia Oghenevwoke Stephen***

***Department of Statistics Federal School of Statistics***

***Enugu- Nigeria***

**Preface**

*Mathematics is peculiar.*

*Mathematics is a peculiarly demanding method of inquiry. It is one of very few human studies that always has only one correct answer. A person trained in its ways can tell with absolute certainty when he has found it. There is no room for* *personal opinion on what is truth.*

*Mathematics is a peculiarly engrossing pastime. Working on a new problem can provide days of anxiety, with those involved sometimes neglecting social response and even neglecting eating and sleeping for hours or days at a time. But when that answer is finally found, the anxiety turns to elation and a time for celebration. Mathematicians are the only professors who use terms like “elegant” and “beautiful” to describe their solutions.*

*Mathematics is a peculiarly rewarding part of any career. Students of fields other than engineering, the hard sciences and economics, often object to their teachers, “I’m never going to use this stuff, why do I have to learn it?” But the answer is easy to provide and documented by statistics. Each college course passed adds a certain amount to the student’s annual salary for his entire life. But each mathematics course passed adds an amount of between two and three times the size of the average course. Whether or not a career ever solves an equation or graphs a parabola, a student of mathematics learns how to think. He earns more money because he is worth it. He produces more in a shorter time, both on the job and in his personal life. By any measure of success, mathematics is worth the time and effort it takes to master.*

*Mathematics is a peculiarly accurate field of study, requiring detailed fine-tuning of its organization and order of presentation. Chapter 3 cannot be understood until chapter 2 is mastered, and chapter 2 requires chapter 1. Indeed, this is why the order of courses in universities around the world is almost identical, more than almost any other subject area. And this is why there must be an entrance examination, such as JAMB, or WAEC, or in the USA, the SAT. They must know a student is ready.*

*Mathematics is a peculiarly challenging subject to teach. Everyone “knows” that there are two kinds of people, those gifted in mathematics, and those who suffer from lifelong mathematics anxiety, who have been forced to order their lives away from any computation, often even simple arithmetic. But the studies are clear as to the cause of this. The problem seems to revolve around one or two teachers encountered by the student during adolescence. The teacher who approaches the subject with the necessary patience, and faith that any student can do it, produces a steady stream of “mathematically gifted” pupils. The teacher who demands results and treats those who need time to understand as inferior, produces the steady stream of mathematically anxious multitudes. One does not “teach mathematics”. One provides the joy of learning something so peculiar, and then waits for time and promises to motivate the student to learn it for themselves.*

*Mathematics is peculiar in how it contributes to the progress of growth. In mathematics, one either understands it, and it is trivial, or, one does not understand it, and it is impossible. Understanding is growth, and in the mind, growth is progressive understanding.*

*Mathematics will never become any less peculiar. But I am pleased to announce, with the publication of this work, it has become a little less demanding and challenging, and thus potentially more rewarding. Mr. Edeh has organized the content well, written solutions that encourage study, and best of all, illustrated the teaching methods that result in the “steady stream of gifted pupils”. By arranging the material to facilitate the passing of the mathematics college entrance exams, he has made entering a rewarding career path that much easier. It is an honor to have been asked to proofread the work and to write the preface.*

***Ken Behrens.***

***BA/MS (Math major/Music minor) — (De Paul University, Chicago)***

***ABD – (University of Nebraska, Lincoln)***

**Definition of Symbols/Acronyms used in the Book**

**• D.A.T:** Divide all through

**• D.B.S:** Divide Both Sides

**• TSOB:** Take square root of both sides

**• π:** Pi (3.14159…)

**• ⇔:** Bi implication (Implication that goes both ways)

**• ⇒:** Implies that

**• ∈:** Is a member/ Belongs to

**• ℤ:** Integers (Numbers that are whole numbers, which can either be negative or positive or zero)

•**ℕ**: Natural numbers ( numbers that are whole, and not negative)

•**ℝ:** Real numbers ( Numbers that can be located on a number line)

**• ≡:** Equivalence/Identity. (Shows that an expression is **always** equal to another)

• **CBTB10:** Convert both sides to base 10

•**CM:** Cross Multiply

•**∨**: Or

•**∧**: And

• **SOF:** Subject of Formula

**• SBS:** Square both sides

• **RHS:** Right Hand Side

• **LHS:** Left Hand Side

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**Chapter 1**

**Definition of Subject Matter**



If you get a balance scale, and place Kg of tins of tomato paste on the left hand side of the scale, and get oranges on the right hand side of the balance scale, the only situation where one side will not outweigh the other, is when they are of equal mass (The oranges altogether weigh ). That’s “**EQUALITY”**

What if another orange, may be, of is added to the right hand side, making them oranges What will happen? You will notice that the RHS will outweigh the LHS. That is “**INEQUALITY”.**

You can decide to call tomato pastes “T” and Oranges “R”. *(“R” was used instead of “O”, for it not be mistaken for “Zero – ”).*

With that,

If you take to the other side, i.e., placing all the oranges on the side where the tomato tins are, you will have Zero/Nothing on the RHS.

That is **(This is an equation).**

Mathematically, an equation is a formula that expresses the equality of two expressions, by connecting them with the equals sign “=”. Also, it is any well-formed formula consisting of two expressions related with an equals sign.

You can simply put it that equations are expressions connected with an “equality” sign.

What then is an expression?

An expression is any variable or element that is not connected with equality sign.

**is an expression but**

**Is an equation**

**is an expression while**

**, is an equation.**

Whenever you are solving an equation, what should come to your mind is, “what will be the value of this variable, for it to be equal to the number/expression at the other side of the equation?”. When you have: no number on earth will 2 be subtracted from, to give aside “” itself. Automatically, 2 becomes the only Solution to the equation.

If you have the question should be, “what is that number that if I take its square, i.e. multiply by itself to give me 36?” The numbers are 6 and -6 because, is and is

So, the answer(s) becomes

Consider the equation; and .

You will still try out the numbers such that when you take the sum, it will give you 6 but if you take their difference, the answer becomes 2.

The 2 numbers are 4 and 2. Why?

But there are equations that you can’t quickly guess the roots, that is why we have methods/approaches used in handling them, which you will meet as you journey through the chapters of this book

**KEYPOINT:** Once an equal sign is there, it is an Equation. It should not be confused with Identity symbol. Identity symbol is used when expression on both sides are equal for all values. It is always true.

**Examples: ,**

For all the values of, the expressions at the RHS and LHS remains equal (always).

**Types of Equation**

There are so many types of Equations in Mathematics. They are;

1. Quadratic Equations (2nd degree)
2. Linear Equations (1st degree)
3. Cubic Equations (3rd degree)
4. Quartic Equations (4th degree)
5. 5th degree to nth degree Equations
6. Trigonometric Equations
7. Transcendental Equations
8. Indicial equations
9. Radical Equations
10. Logarithmic Equations
11. Hyperbolic Equations
12. Combinatorics Equations

Etc….

But, we are not going to be exceeding the O’level specifications, which are;

Linear, Quadratic, Simultaneous-Quadratic, Trigonometric, Radical, Indicial, Logarithmic, Combinatorics/Factorial, Fractional Algebraic, Polynomial, Number base, Modular, Modulus (Absolute Value) Equations.

**Chapter 2**

**LINEAR EQUATIONS:**

*This is the easiest type of equation, when you talk of O’level specifications. It doesn’t have any other power higher than 1.*

It can come in so many ways,

1. **1 system linear Equations**
2. **2 system linear Equations (Simultaneous Equations)**
3. **Diophantine equations**

In one system, you solve for one variable only. You must be conversant with change of Subject of Formula for you to have a proper mastery of this type of equation.

Diophantine equations remains the toughest in this system of equation (we won’t be discussing that).

**Example 1: find the value of**

**Solution:** Shift 8 to the other side, when crosses the sign of Equality, it will change to;

Divide both sides by the coefficient of, which is.

You have,

**Example 2: Find**

**Solution:**

Expand

Now,

***C.L.T:***

***DBS by 5:***   ***⇒***

**Example 3: find**

**Solution:**

***Cross multiply:*   ⇒**

**Expand**

**C.L.T:**

**Cancel minus on both sides:**

**Example 4: find a**

**Solution:** Expand:

***Simplify & C.L.T:***

*⇒*

**D.B.S by 5:**

**Example 5: find y**

**Solution:**

**Cross Multiply:**

**Example 6: find**

**Solution:** . The LCM of 6 and 12 is 12, so it is convenient to multiply all through with 12.

⇒

**Expand:**

**Example 7: find x**

**Solution:** Multiply all by the LCM of which is 60.

**Example 8: Solve (WASSCE)**

**Solution:** The LCM of 6 and 4 is 12. If you multiply throughout with 12, you obtain

**Example 9: Find y**

**Solution:** Divide both sides by 0.25 ⇒

**Example 10: Find the value of x which satisfies the equation;**

**Solution:**

**Questions to try: Solve forin the following equations.**

**•**

**•**

**•**

**•**

**•**

**•**

**Chapter 3**

**2 SYSTEM SIMULTANEOUS EQUATIONS**

***(Simultaneous–linear Equation)***

This is the type of equation where you will be asked to find the value of 2 variables/ unknown. You are to solve them one after the other (Simultaneously)

**There are 4 methods of doing this:**

**1. Substitution Method**

**2. Elimination Method**

**3. Graphical Method**

**4. Subject of Formula Method**

**4. Matrix Method.**

(But we are only going to be using Elimination Method & Substitution Method in this chapter.)

**Example 1: Consider this system of equation; and find the values of x and y.**

**Solution:** Now, you must first of all understand what “Elimination Method” is all about before tackling this problem.Here, you must make sure that a variable in the first equation, must be present in the 2nd equation. In this case, you either choose to add both equations or subtract one from the other.

We are having x and y with coefficients of 1, so we don’t have to do much work.

**Step1: Subtract Equation 2 from Equation 1**

(If you check, we eliminated x from the first solution, the next will be to eliminate y)

To eliminate y, you add both equations;

With that

**Example 2: If, find the values of x and a*.***

**Solution:**

Right here, we can’t just add or subtract the equations because, they don’t have similar coefficients.

What we will do is that, we will multiply equation 2 by the coefficient of x in equation 1 and multiply equation 1 by the coefficient of x in equation 2. This is because we can’t to eliminate x. **coefficient of x in equation 1 is 2.**

So, ⇒

The coefficient of x in equation 2 is 3

So, ⇒

If you check both equations, you will realize they are having a variable in common **(which is 6x)**

Here, you can subtract equation 2 from equation 1.

Cancel “minus (-)” on both sides,

11a = 26 ⇒ ≈ 2.363636…

Going back to one of the equations,

**DBS by 2;**

With that, a = and x =

**Example 3:**

**Solution:**

A very easy way to solve this equation and its kind, is to add both equations.

(Reason is that both equations have a variable with different signs, i.e, one is positive and the other is negative)

In substitution,

DBS by 5:

Now,

**Example 4:**

**Solve for x and y respectively: (UTME)**

**Solution:** We will be using Substitution method.

From I, let’s make x subject of formula,

Substitute x = in equation II:

**C.L.T:**

**DBS by 6:**

Since y = –1, where ever you see “y” in any of the equations, you will call it “-1”.

Choosing quation II,

Hence,

**Example 5: Find the value of x and y in the simultaneous equations:**

**(UTME)**

**Solution:** One of the ways of handling questions of this nature is making a variable, subject of formula in equation and substituting into equation II, but it will take us to “QUADRATIC EQUATIONS”. Since we haven’t done Quadratic Equations, we will make use of Algebraic identities.

From I, If you square both sides, you get

But from II, we will now have:

⇒

Remember,

Therefore,

If we take +9,

Now, from I,

If you add IV & I, you have;

In substitution into I,

With that, and

If we take -9,

From I,

If you add V and I, you have;

In substitution into I,

In conclusion, when and when

**Example 6:**

**Solve the simultaneous equations: (UTME)**

**Solution:** Let will be and.

The equation will now be:

Make “k” subject of formula in equation I:

Substitute in equation II.

Substitute in any of the equations, choosing equation II.

Recall that

Also,

**Example 7: Solve for x and y in the simultaneous equations: (WASSCE)**

**Solution:**

From I, , where ever you see in II, call it

⇒

Where ever you see “y” in any of the equations, call it “1”. I will work with equation I.

With that, and

**Questions to try: Solve forin the following equations.**

**•**

**•**

**•**

**•**

**Chapter 4**

**QUADRATIC EQUATIONS**

A quadratic function is a Polynomial function with one or more variables, in which the highest power of a variable doesn’t exceed the power of 2.

Consider these functions; etc., the powers of must not exceed power 2

**It is an algebraic equation of the second degree in x.** The quadratic equation in its standard form is, where are the coefficients, x is the variable, and c is the constant term,. When writing a quadratic equation in standard form, the term is written first, followed by the x term, and lastly, the constant term is written.

Kindly note, after solving a quadratic equation, you must obtain two roots (solutions).

In a situation where it is having one root (the discriminant is zero), you say it is “that number” twice.

Example: , etc.

**Types of Quadratic Equations:**

There are 3 types of Quadratic Equation. We have;

**Standard/Adfected Quadratic Equations.**

**Quadratic Equations with constant term equal to zero.**

**Pure Quadratic Equations.**

**Adfected Quadratic Equations**: This is otherwise known as Standard Quadratic Equations. It is the type of Quadratic Equation that is in the form of **ax²+bx+c = 0.**

**Quadratic Equations with Constant term equal to zero:** Just as the name implies, this type of equation lacks the constant term. It comes in form of **ax²+bx = 0**

**Pure Quadratic Equations**: They are those Equations that comes in form of

**ax²+c = 0.** It is arguably the easiest type of quadratic equations.

**Forms of Quadratic Equations:**

Likewise the types, a quadratic equation can also come in 3 forms.

**Standard form:**

**Factored form:**

**Vertex form:**

METHODS OF SOLVING QUADRATIC EQUATIONS

**Square root Method**

**Factorization Method,**

**Completing the Square Method,**

**Formula Method,**

**Graphical Method,**

**Calculus Method,**

**PO SHEN LOH’S Method.**

We are going to be making use of Square root, Factorization, Formula Method & Completing the Square Method in this chapter.

Before that, let’s consider some of the questions below.

**SQUARE ROOT METHOD**

**Example 1:** . **Find the value of x**

**Solution**:

(When minus crosses the sign of equality, it changes to plus)

(*Take the square root of both sides)* ⇒

**Example 2: Find x**

**Solution**:

*(Take square root of both sides)* ⇒

Note: It is ± because Square Function is an even Function.

i.e., f(-x) ≡ f(x). Remember, when it is not an equation, is 2, is 5 and = 10. E.t.c.

**Example 3: Find the value of x**

**Solution:** ⇒ ⇒

**DBS by the Coefficient of** , which is 2 ⇒

**Example 4: What value of “a” makes the equation true?**

**Solution:** Factorize

**Example 5: Find x**

**Solution:**

**DBS by 4:**

TSOBS; Note:

**FACTORIZATION METHOD IN SOLVING QUADRATIC EQUATIONS**

**Factorization method** is used to find the value of the variable in question, if only the given expression can be factorized, over rational expressions/numbers.

**Note**: **All Quadratic Equations are factorable, just that if the solution to the equation be, a complex number; surd, it will be very hard to factorize, especially if we are considering the Scope of Senior Secondary School Certificate Exam (SSCE)**

The steps followed in the Factorization of Quadratic expressions are also used here. It means that if you can factorize a quadratic expression, then solving a quadratic equation will not be a hard nut to crack.

**Keep this in mind:** “If ,then either “”. Do you know why? It is because .

Similarly, if you have , just know that x = a & x = b. This rule is not applied if (x-a)(x-b) = c, where c is any constant.

We will apply these established rules in the following examples:

**Example 6**: **. Solve for x**

**Solution:** Notice that this equation doesn’t have a constant term “c”. At such, we factorize without any manipulation.

x is common;

you can make it unique by rewriting it to be

If AB =0, then either A =0 or B =0 or AB = 0

**Example 7**:  **find t**

**Solution:** You have to look for 2 numbers, such that when added will give -7 and when multiplied will give 10. The numbers are and

After finding the two numbers, the job is done. You will have something like: , where r and s are -2 and -5 respectively.

And

And

**Example 8: If Find a**

**Solution:** Two numbers that if added will give 4 and if multiplied will give 4, they are 2 and 2.

Twice

**Example 9: Consider find the values of y.**

**Solution:** Two numbers that their sum is 10 and their product is 24 are 4 and 6.

**Example 10: If find x**

**Solution:**

Two numbers that their sum will give 12 and their product is 27 are 3 and 9.

**But, what if the Coefficient of x² is not 1, like the ones we solved?**

**Example 11: When , find x**

**Solution:**  You will take the product of the first term “a” and the last term “c”.

A very big question: What are the numbers whose sum is 4 and product is 3×-7 (-21)

**The numbers are 7 and -3*.*** You will now have;

and

**Example 12: find k**

**Solution:** Two numbers whose sum is -3 and product is 4×-22 (-88) are 8 and -11

**Example 13:**  **find x**

**Solution:**

*Two factors when added, gives “+40” and when multiplied, gives “-84”.*

*They are 42 and –2*

**Factorize**: 6 ⇒

**Example 14: find x**

**Solution:** This is solvable using square root method, but it is necessary to make it clear that, there are more than just one method to handle questions like this. We can express the above expression as a difference of two squares, from there we factorize.

was chosen because = 6

**Example 15:**  **what is the value of x**.

**Solution:**

Since 4 is common, you can factorize out 4.

If you divide both sides by 4, nothing changes. The equation remains unchanged/valid.

⇒

**There is a very short technique we should take note of. It is easier that the traditional method of solving Quadratic Equations using Factorization.**

**Example 16: If** **you have something like:** ***,***

*Pick the coefficient of the first term, which is 2 and multiply it by the Constant term.*

You have:

If we factorize, we are going to get:

*Now, divide the numerical values in the bracket by the “2” you used in multiplying the Constant term.* You will have:

⇒

**Example 17: find x**

**Solution:** *Take the coefficient of the First term “5” and multiply the Constant term.*

**Factorize:**

*Because we multiplied with 5, I will divide with 5 this time around. I will have;*

⇒

**Example 18: Find the roots of the equation (UTME)**

**Solution:**  is

If we factorize, we obtain;

Divide the constants in the linear factors by 10;

**Example 19: Solve the quadratic equation (New Concept Mathematics for Senior Secondary schools III)**

**Solution:** Multiply -5 by 2 ⇒

Factorize;

Divide the constants in the linear factors by 2;

**Example 20: If find p.**

**Solution:**  ⇒

Factorize;

Divide the constants in the linear factors by 2;

and

It is noteworthy that, not all equations can be factorized (within the scope of O’Level). Some of those equations that are not factorable, we solve them with the use of FORMULA METHOD.

**FORMULA METHOD:**

*This is what most regard as “Almighty formula”. It should, as a matter of fact be known the Formula doesn’t possess any trait of mighty formulae.*

The Formula says that, when you have an equation of the kind

The solution is, . Provided a≠0

*What we are mostly going to be making use of, is substitution.*

**Questions to attempt:**

**Example 21:**

**Example 22:**

**Example 23:**

**Example 24:**

**Example 25:**

**Example 26:**

**Example 27:**

**The positive root of t in the following equation,**

**Correct to 4 places of decimal, is? (UTME)**

**Solution 21:**

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

Here, a=1, b=2 and c=1

The formula is x =

⇒

⇒

But because it is a Quadratic Equation, we say, “–1twice”

**Solution 22: and**

⇒

⇒

Note:

;

(Only make use of Calculator value, i.e. approximate, when asked to)

**Solution 23:,**

Formula is x

**Solution 24:** ;

or

**Solution 25:**

⇒

Or

The method used in solving this particular question can be “too long”, for some students. Some may want a method/approach that is not lengthy. The reason why it is as if it is lengthy is because of the presence of fractions that we resolved on the course.

One can as well say:

Multiply the equation through out by the LCM of their denominators.

The LCM is

From this point, you can use the quadratic formula.

How fast? Lol.

Note: and means the first and second value of x, respectively.

**Solution 26: ;**

⇔

**2nd method:**

*Multiply through out by 6:*

Using quadratic formula,

**Solution 27:**

But because we are to solve for the positive root, will be considered.

**Do these:**

**F.**

**G.**

Correct to 5 decimal places where necessary.

**Chapter 5**

**SIMULTANEOUS-QUADRATIC EQUATIONS**

This topic would have been treated under Simultaneous equations but because it is involving Quadratic equations, we decided to place it after Quadratic equations.

You should see Introduction of Simultaneous Linear Equations, in order to know what this chapter is all about.

Always keep in mind that matrix method cannot be used here, because Matrices is under *linear Algebra****.*** Elimination method too, can’t work here. The 2 known ways are Substitution and Graphical. We will be making use of Substitution method.

**Example 1: Solve, (UTME)**

**Solution:** From

In substitution into “

Further more, twice

But

Hence, when

**Example 2: Find the values of x and y, in the equation:**

**(UTME)**

**Solution: If**  then

In substitution,

Recall that,

Also,

We conclude that, when and when

**Example 3: Solve,**

**Solution:** From

In substitution:

**D.A.T by**

Because

Also,

With that, when when

**Example 4: Solve for x and y**

**(New Concept Mathematics For Senior Secondary school III)**

**Solution:**

Recall that:

Divide equation II by equation I;

From

In substitution,

Recall that

Conclusion; when

**Example 5: Solve,**

**(New Concept Mathematics For Senior Secondary schools III)**

**Solution:**

From II,

From I,

In substitution into II,

Multiply throughout with 2;

Since when

Also, when

Hence, when when

**Example 6: what values of x and y satisfies the equation?**

**(New Concept Mathematics For Senior Secondary schools III)**

**Solution:**

In substitution,

Using quadratic formula;

To find x,

One can conclude that when

**Example 7: find a and b**

**Solution:** Whenever you have A=B=C, It means that, A = B, A = C and B = C.

So,

And

From IV,

Substitute b into III;

Since

Also,

Hence, when when

**Questions to try:**

**• Find a and b, if and**

**• show that is less than 3**

**•. If , find to 4 decimal places.**

**• find the two values of k.**

**• If a² + b² = 16 and 2ab = 7, find all possible values of (a – b). (UTME)**

**Chapter 6**

**EXPONENTIAL (INDICIAL) EQUATIONS**

An exponential function is defined by the formula, where the input variable x occurs as an exponent.

If you have something like

Is the base and is the exponent/power.

For you to be able to handle questions under this chapter, you must either; **Make sure that the base in the RHS = LHS or that the Power of RHS = LHS.**

In a situation where you cannot either make the bases or the powers equal, you introduce Logarithm to both sides.

Before now, we must have treated “INDICES”. This chapter will only handle the equations aspect of it.

Remember, for where

*Consider this equation****:***, find the value of x.

*In this case, what you should be trying to establish is for the base in LHS to be same with that of RHS...*

If you have, **“a” will cancel “a”, leaving you with x = 3**

If you have ,“**x” will cancel “x”, leaving you with b = 2.**

With that, will cancel

**x = 2**

What of “

*Establish a same base:*  ***⇒***

⇒

*Easy right? Let’s try another one.*

**Example 1: find the value of**

**Solution**:

**3 will cancel 3;**

**Example 2: , Find the value of a**.

**Solution:** *Since, 2, 4, 8 can all be expressed as numbers with a base 2,*

⇒

**(Review rules of Indices)**

2 will cancel 2 ;

**Example 3: , find n.**

**Solution:**

**Note:**

**⇒**

**⇒**  ⇒

**:**

**Example 4: , find the value of x**

**Solution**:

**Divide both sides by**

**Remember that, once they are of same power.**

; 1 can as well be written as

; **cancel on both sides,** x = 0

**Example 5: What is the value of x, satisfying the equation**  **(UTME)?**

**Solution:** From Indices’ rule 2,

Then,

**Cancel 2 at the two bases;**

**Example 6: Solve for x in (UTME)**

**Solution:** DBS by 8;

**Cancel -2 at the powers:**

**Method II:**

Cross multiply:

DBS by

Note: Indices often ignores /Other Solutions to equations, there by picking only the principal solution.

But since we use d square root method (either of quadratic equations), it will give us two answers. It is noteworthy that “-10” world perfectly into the equation.

**Example 7: If find x (UTME)**

**Solution:**

**Cancel 2 at the bases;**

**Example 8: and find the value of x and y.**

**Solution:**

This is an Indicial Simultaneous equation.

The indices will be resolved, the Simultaneous- Linear Equation will be left as an assignment to the reader.

First things first, express every number, such that it will have a base of 3.

And

;

Cancel 3 at both bases of the 2 equations

**(Solve it on your own)**

**Note: the bases must not be the same. Equation 1 can have a base of 5 while Equation 2 is having a base of 3. It must not be always of the same base.**

**Example 9: Solve for y in the equation below:**

**Solution:**

**Cancel 7 at both bases:**

**C.L.T:**

Cancel the negative signs on both sides or divide by “-1”;

**Example 10: find the value of x**

**Solution:**

(Any number raised to power of 0, aside 0, is 1)

**Cancel 81 on both sides:**

**Example 11: find x to 2 decimal places.**

**Solution:**

**(Cancel the bases):**

**Solving for x, add 4 on both sides i.e. to complete the square;**

Take square root of both sides:

to 2 d.p

**Example 12: Solve the equation: (WASSCE)**

**Solution:**

Make every variable to have a same base. i.e., “2”

Note:

Cross multiply

Since they have same base, you can now multiply. Remember that;

(First law of indices).

⇒

Cancel the bases:

**Divide both sides by “-8”**:

**Example 13: Solve for x, if**

**Solution:** Convert all to a base of 3.

Cross multiply;.

*Cancel 3 at the bases* ⇒

**Example 14:**

**If**  **find the value of a (WASSCE adapted)**

**Solution:**

Resolve the denominator:

Make every variable to have a base of 5:

Cross Multiply:

Resolve the power:

Cancel 5 at the bases,

**C.L.T:**

DBS by 3:

**Example 15: find the value of k that makes the equation true.**

**Solution; If you check, this question is different from the ones we have been solving. What you have to do first is to Identity the type of equation it is. Here, the question is a “Quadratic-Indicial” equation. It is because, method for solving quadratic equations is needed here.**

For easy computation, let

Familiar?

Using factorization,

twice

Recall that therefore,

Cancel 6 on RHS and LHS

**Example 16:**  **solve for x**

**Solution:** Cross multiply:

*Let ⇒*

**We are not going to work with the negative result because the value exists in the complex plane, i.e., it is not a real number.**

But So,

**Example 17:**  **find p**

You can move the power at the denominator to the numerator.

If the power at the denominator is negative, it will change to positive. If it is positive, it will change to negative.

**Example 18: Solve the equation (WASSCE)**

**Solution:**

**Cancel 2 at the bases:**

**Example 19: If find the value of (WASSCE)**

**Solution:**

**Example 20: Given that , find in terms of t (WASSCE)**

**Solution:**

Raise both sides to the power of negative 1, i.e., “-1” ⇒

**Multiply both sides by**

**But**

**Example 21: If if evaluate (WASSCE)**

**Solution:**

With that,

**Example 22: If find the value of x. (UTME)**

**Solution:**

**Cancel 5 at the bases;**

**Example 23: If find the value of (WASSCE)**

**Solution:**

Divide both sides by

**Example 24: Solve the equation (WASSCE)**

**Solution:**

Let

But. Also,

**Example 25: Solve for x if**

**(500 Boosters Problem Solving in Mathematics)**

**Solution:**

There some equations that you cannot express both sides, to have same base. Such equations, we handle them with the help of “logarithm”. We apply Logarithm to both sides in order to bring the exponents down to the base level. You should know that this is not under Logarithmic equations, as you are the one involving/applying the logarithm your self. Logarithm here, is just a tool.

**Example 26:**, **find y**

**Solution:**  , add Logarithm to bothsides;

DBS by

**Example 27: what is the value of x?**

**Solution:** Add Logarithm to both sides;

*Remember, Logarithm with no base attached to it, is to the base of 10.*

DBS by

**Example 28: If , find x**

**Solution:**  Add Logarithm to both sides; (I will add Natural Logarithm this time)

Which is equally

**Example 29:**  **Find x**

**Solution:**

Let

Multiply all through with

But

Approximately,

**Questions to try. (Solve for the unknown):**

**Chapter 7**

**LOGARITHMIC EQUATIONS**

The logarithm of a positive real number x with respect to base b (is the exponent by which b must be raised to yield x. In other words, the logarithm of x to base b is the unique real number y, such that when you have .

The logarithm is denoted “” (pronounced as “the logarithm of x to base b”, “the base-b logarithm of x”, or most commonly “the log, base b, of x”).

An equivalent and more succinct definition is that the function is the inverse function to the function

On a simpler note, Logarithmic equations are those equations involving Logarithmic properties.

There are 3 types of Logarithm in Mathematics, we have the;

* Common Logarithm(Briggsian Logarithm, decadic Logarithm)
* Binary Logarithm
* Natural Logarithm (Napierian Logarithm)

Common Logarithm makes use of base 10 while Natural Makes use of base “e”, where e ≈ 2.71828…

Note: you are free to make use of either functions, but you should bear in mind that they are not equal to each other.

If you have something like: your question should be “what base is the Logarithm having”

In a situation where no base is stated, you assume the base to be “10”

It will now be:

**The “10” will automatically become a base number that will raise “3”**

**Example 1: Find x**

**Solution;**

**Example 2: Solve “”**

1. x = 3, b. x = 4, c. x = 8 , d. x =

**The correct option is b, x = 4**

Why?

**Example 3: Find the value of k, if**

**Solution;** the base is k, the next thing is to get rid of the base.

When we take square root of both sides, we will have; ⇒

Taking the positive value, k = 3

**Example 4: find the value of x**

**Solution**:

**Since, they are of same base, adding both should be the next step.**

⇒

To 7 decimal places.

**Example 5:**, **find the value of x**

**Solution:** Add Logarithm to both sides:

**(**Because it is in base 10, Log 10 = 1)

Take square root of both sides

**Example 6: Solve for x**

**Solution:**  Add Logarithm to both sides ⇒

Divide both sides by ;

**Example 7: If find x**

**(New Concept Mathematics Book 3 (SSIII)**)

**Solution**: Because the conjunction between them is “–“, we will divide.

**Remove the Logarithm:**

**Cross multiply: ⇒**

**C.L.T; ⇒ {– will cancel –}**

DBS by 28

**Example 8: . What is the value of x**

**(New Concept Mathematics Book 3 (SSIII)**

**Solution:**

**Cancel Logarithm on both sides:**

**Cancel 6 at both powers:**

**Example 9: (WASSCE)**

**Solution:**

**DBS by 3**:

**Example 10:, Find the values of m and n.**

**Solution:** Let = , while =

**Solving Simultaneously,**

**Recall that**  so

**And** i.e.

**Example 11: Solve for x in:**

**(New Concept Mathematics Book 3)**

**Solution:**

Did you notice that 2 is not having any Logarithm? What are we going to do?

Ask yourself what Logarithm to a base of 10, will give you 2?

You will have something like:

So, it means that

**We can now say:**

**Cancel “Logarithm to base 10” on both sides,**

**Example 12: find the value of x to 1 decimal.**

**Solution:**

**:**

**Example 13:**  **find x**

**Solution:** Convert “1” to a Logarithm in base 7, which is

Cancel Logarithm to base 7 on both side

Solving the quadratic Equation,

**Example 14:**  **Find x (NABTEB)**

**Solution:**  Note: is same as

But base cannot be negative, we say

**Example 15:**

**Express p in terms of x and q**

**Solution:**

**Example 16: If**  **what is q (WASSCE)**

**Solution:**

**Example 17:**  **find x**

**(Hidden facts in SSCE Mathematics)**

**Solution:**

Cross Multiply & Expand:

CLT:

**Example 18: find the value of x (UTME)**

**Solution:**

Square both sides:

**Example 19: what is x (UTME)**

**Solution:**

**T.S.O.B.S;**

Going with the positive value, you get

**Example 20: express x in terms y and a (NECO)**

**Solution:**

Cancel Logarithm to base “a” on both sides;

**Example 21: find x**

**Solution:**

N.B:

Multiply the powers of both sides by -1:

**Questions to try.**

Solve for and in the following:

1. **if**
2. **Write the equation in a form not involving Logarithm.**

**Chapter 8**

**RADICAL EQUATIONS**

In Mathematics, radical properties are those properties having nth root. It could be Square root, cube root, 4th root etc., they are equations, in which variables are under radical.

It therefore means that you introduce nth power of a function with an nth root to get it eliminated.

Say

Remember, any number/variable under the square root sign, is called “**RADICAND”**

**Keep the following in mind**:

• A radical expression is an expression that contains a radical. Where the “radicand” is under the nth root function.

• No radicands have perfect nth powers, other than 1.

• No radicals appear in the denominator of a fractional expression. **[FOR MORE EXPLANATIONS, VISIT “SURD”]**

**Example 1**: **If , find x**

**Solution:**

*Here, the Function is square root, so, we will introduce square to bothsides.*

**Example 2: If**  **solve for the value of x.**

**Solution:** Square both sides ⇒

**Example 3:**  **find the value of x.**

**Solution:** Square both sides ⇒

Divide throughout by 2

⇒ ⇒

Shift x to the other side

Square both sides

(Cancel on both sides)

CLT

**Example 4: find x**

**Solution**:*Shift the one with no radical property to the right hand side****.***

**Square both sides**

12 is not equal to 9, so 6 is a valid solution.

You should have in mind that, when ever you are solving equations like this, it will always yield one valid solution and an extraneous solution.

**Example 5: If Find the value of a**

**Solution:** Since we’ve learnt that no radicand will be at the denominator of a fraction. When we are talking of radical equations, it will be wise to rationalize and get rid of fractions

Now, expand

**[**  cancels  **] ⇒**

**Rationalize & DBS by 2:**

**Example 6:**  **find x**

**Solution:**

In this case, we are going to square both sides ⇒

***D.A.T by 2:***

**Take x to the right hand side:**

**Square both sides:**

**Will cancel x²**

**Example 7:**. **Find the value of y that makes the equation true.**

**Solution:**

**Square both side:**

**Square both sides again:** ⇒

CLT:

Using Quadratic Formula,

Let’s check whether we are correct, the original equation is

Checking ...

….

(False/Not balanced) ⇔ ,which is not 6

Testing

(True/Balanced), so the value of x is

**Example 8:**  **find the value of x**

**(500 BOOSTERS problem solving in Mathematics)**

**Solution:**

**Cross Multiply**:

**Expand**:

**Divide both sides by** “

**Example 9:**  **find the value of x**

**Solution**: **[C.M]**

**S.B.S:**

**[8x² will cancel 8x²]:**

**[C.L.T]:**

**Example 10:**  **find the value (WASSCE)**

**Solution: Rationalize** gives

**(Visit Manipulation of Surds)**

**Now,**  ⇒

**[DBS by]**:

**[C.L.T]:**

**Example 11:**  **find the value of**

**Solution:** There exists a difference between equality and equivalence. In that same way, equality of surds is different from equation of surds. This particular question was introduced to clear a doubt. You can have a question like; find the value of Now, it is very wrong to use equality sign because this question is actually not easily solved, over OLEVEL if it were to be “equal sign”. It is a “Diophantine” equation. An equation where we are to solve for 2 variables using one equation. It is just like saying, x + 2y = 10, find x and y. Of course there are so many values of x and y. We can have the solutions paired as,

The values are endless since there is no restriction placed on it.

**is equally going to have so many solutions, if it were to be sign of equality.**

Itwill be better to write it as

We now equate

To to

Will be 1

**Back to business,**

**Re-arrange:**

So,

**Example 12: If Find the positive value of x (WASSCE)**

**Solution:** Square both sides:

**T.S.O.B.S:**

**Example 13: ,find k (WASSCE)**

**Solution:**

**Example 14: Given that, find k (WASSCE)**

**Solution:**

**Example 15: Express in the form of (UTME)**

**Solution:** Rationalize;

**Example 16: Solve the equation (UTME)**

**Solution:**  Shift “-1” to the RHS and square both sides,

Shift the expression with radical property to one side:

Square both sides:

**Example 17: Find the solution of the equation**

**Solution:** Shift the expression with radical property to one side:

Square both sides ⇒

Checking whether 9 fits into the equation, (Balanced)

Checking whether 25 fits equally, (Balanced)

So, 9 and 25 are the solutions to the equation.

**Questions to try:**

* **find x**
* **Find the value of y**
* **show that x is less than 1**
* **What is the value of**
* **Given that find (WASSCE)**
* **Solve the equation (UTME)**

**Chapter 9**

**FRACTIONAL ALGEBRAIC EQUATIONS**

In mathematics, a fractional algebraic equation is an equation where the unknown variable appears in the denominator/numerator of a fraction. For example, the equation can be considered a fractional algebraic equation. To solve a fractional algebraic equation, you need to first multiply both sides of the equation by the denominator of the fraction on the left-hand side, so that the unknown variable is not in the denominator. This gives you the equation , which can then be solved using standard algebraic techniques.

The outcome (After getting rid of the denominator) of this very topic, has been dealt with, in the previous chapters. (**Linear/Quadratic Equations)**

**Example 1: Find p.**

**Solution**: Note that; one among the rules governing Algebraic Equations is that; “*whatever you do to the right hand side, do it to the left hand side, except multiplying and dividing by zero (0)”*

We can’t just start and use just any number. No! We must make sure it is going to aid the computation. i.e., (**MANIPULATION)**

LCM of 2 and 3 is 6. I can choose to multiply both sides by 6

*(You can cross multiply if you do not want to multiply by the LCM)*

**Example 2: , solve for y.**

**Solution:** If you look at this very equation, there is something strange and unique as well, about the denominator. It has 3 different denominators right? Do not fret!

The LCM of and is. Reason is that you can’t repeat a factor.

If you have something like the LCM of the answer is just and not

Back to business. Now, multiply both sides by the LCM

will cancel While will cancel

2 d.p

**Example 3: , find the value of x**

**Solution:** The LCM of x and 9 is 9x. So,

(x will cancel x and 9 will cancel 9)

**TSOBS:**

**Example 4: find x**

**Solution:** *The first thing to do is to open the bracket (Though you can choose to divide both sides by 6)*

*⇒*

DBS by 3

**Example 5:**   **what is the value of x?**

**Solution: *Cross Multiply***

**Expand:**

**[C.L.T]:**

**[DBS by “-16”]:**

**Example 6: find x**

**Solution: {Cross Multiply};**

**Divide both sides by (x+6):**

Expand ⇒

CLT ⇒

**Example 7: find the value of p (WASSCE)**

**Solution: {Resolve the fractions at the denominator}:**

**Cross Multiply:**

**Expand the bracket by distributing the numbers outside the bracket to the ones inside;**

**Cross multiply again:**

**Expand:**

**Example 8: and**  **Find the value of x and y**

**Solution:**  and we will now have: and

Solving that using elimination method (which we’ve explained before),

But, So, and So,

**Example 9:**  **find the value of x and y**

**Solution;** Whenever you have an equation like; a=b=c. It is not a big deal. What you have to know is that is either equal to “b” or “c”, c is either equal to “a” or “b” and b is either equal to “a” or “c”.

You can say, a=c and a=b or even c=b. Their values are all equal.

So, and

The second equation says that ⇒ ⇒

⇒

Now, y = 4. So, ⇒

With that, x = 8 and y = 4

**Example 10: Solve for r if**  **(UTME)**

**Solution:**

Multiply throughout with the LCM of and which is

**Expand:**

⇒

**D.A.T by 5:**

Solving for r,

and

**Example 11: If. , find the value of x**

**Solution:** Cross multiply:

Subtract from both sides & Simplify:

C.L.T:

**Example 12: Solve for x, (WABP)**

**Solution: Cross multiply**:

**Example 13: find y**

**Solution:** Shift “4y” to the RHS and Cross multiply;

Solving for y,

**Example 14:**  **find y**

**Solution:** Multiply both sides by the LCM of and which is

Expand:

**Example 15: If**  **Solve for**

**Solution:** Divide the numerator and denominator by “y”:

Where

**Example 16:**  **Find (WASSCE)**

**Solution:** Divide the numerator and denominator by y:

Let

Cross multiply:

Since

Alternatively, we can solve the problem this way:

Cross multiply:

Expand:

CLT:

DBS by

In conclusion,

**Example 17: evaluate**

**Solution:** If

In divide the numerator and denominator by “y”: where the expression evaluates to:

**Questions to try:**

* **Find the value of x that makes the equation true.**
* **find a**
* **Find the value of x, provided x≠**
* **Find x and y**

**Chapter 10**

**TRIGONOMETRIC EQUATIONS**

Trigonometry is derived from the Greek words, trigonon (triangle) and metron (measure).

It is a branch of mathematics that deals with the study of triangles and the relationships between the sides and angles of a triangle. It’s used to solve problems involving triangles, circles, and other geometric shapes, and it’s also used in fields like engineering, physics, and astronomy. Trigonometry deals with concepts like sine, cosine, tangent, and their inverse functions, as well as trigonometric identities and equations.

A trigonometric equation is an equation involving one or more trigonometric ratios of unknown angles at both sides of an equation. The trigonometric equations contains Trigonometric functions, treated as variables. The angle of x trigonometric functions such as is used as a variable in trigonometric equations. Similar to general polynomial equations, the trigonometric equations also have solutions, which are referred to as principal solutions, and general solutions

Trigonometric equations involves trigonometric functions, such as), and their inverses which can be used to model various types of periodic phenomena, such as oscillations, waves, and rotations.

There are many types of trigonometric equations, and the specific steps required to solve them depend on the form of the equation and the types of trigonometric functions involved.

At the end of your solution, use the periodicity of trigonometric functions to get the general solution to your equation; trigonometric functions have a periodic nature, which means that they repeat over a certain interval. This can be used to find all possible solutions to a trigonometric equation by considering multiple periods of the function.

Do not forget to check your solutions, to check that they are actually valid solutions by substituting them back into the original equation and verifying that both sides are equal.

Solving trigonometric equations can be challenging, and it may be helpful to work through examples and practice problems to gain a better understanding of the process.

In Trigonometric equations, you will be seeing things like;

*Where p, k and c are Constants.*

Inverse of Trigonometric functions are most at times applied on both sides of a given equation. It makes sense until a particular side of the equation is containing more than one variable.

**Notes:**

* *The inverse function of cosine(x)is Arccosine(x) or*
* *The inverse function of Sine(x) is Arcsine(x) or*
* *The inverse function of Tangent(x) is Arctangent(x) or*
* *The Inverse function of Secant(x) is or*
* *The inverse function of Cosecant(x) is or*
* *The Inverse function of Cotangent(x) is or*
* *Under O’level, the values of Sin(x) and Cos(x) are always less than 1. It is only Tan(x) that is greater than one. ( When comparing the basic 3 Trigonometric functions)*

**When solving for the General Solution of a particular trigonometric equation, and we are asked to provide up to 2 solutions to it, it is not farfetched.**

* **The period of Sine, Cosine, Secant and Cosecant function is 2π. It means that the value of sine x and cosx are equivalent every 2π units. π in this case is 180°**
* **The period of Tangent and Cotangent function is π. This shows that every π units, the value of the value of Tanx and Cotx remains unchanged.**

**Sample:** if we have,

To find x, Introduce Sine Inverse (Sin⁻¹) or ArcSine to both sides.

That is,

If we equally have to find x, you introduce sine function to both sides;

In short, if a Trigonometric function is

That is:

***Let’s take a look at some of these questions:***

**Example 1 find x**

**Solution: {Introduce Sine Inverse to both sides}:**

“SHIFT”, then press

On your calculator, press:

4 decimal places.

General Solution: x ≈

**Example 2:**. **Find x**

**Solution**: (Introduce cosine Inverse on both sides)

On your calculator, press

to 4 decimal places .

**General Solution:**

**Example 3: find the value of y**

**Solution: [Introduce Tan inverse to both sides]**

**General Solution:**

**Example 4:**  **, Find the value of y**

**Solution: (**Divide both sides by Cos(y)):

Remember that,

(Divide both sides by 2):

**General Solution:**

**Example 5:**  **Find the value of x**

**Solution: Because Sine function is having “3x” while Cosine function is having “6x”, we cannot easily divide both sides by any. What we have to recall is that, in our basic trig. Identities,**

With that, we can say can also be written as

To verify, check if

Now,

**[Cancel “Cos” on both sides],**

But this method gives only the principal solution, if we are asked to get other solutions , the best thing to do is to rewrite the equation to be

Now,

From and , we can get x = 10° and -30° respectively.

Note: If we expand to, we will get a cubic equation, which is not treated under the Scope of SSCE. Therefore, we only take 10° and -30°

**Example 6:, find the value of B**

**Solution:**

**[Divide both sides by 9]:**

[**Take Cosine inverse of both sides]:**

4 d.p

**General Solution:**

**Example 7: ×find the value of A.**

**Solution:** Recall that

**[Divide both sides by 16]:**

[**Introduce Sine Inverse to both sides**]:

2 d.p

**General Solution:**

**Example 8:**  **The angle (θ) is equal to what?**

**Solution:**

We can recall that

When we divide throughout by , we get:

*It means that where ever you see* *you can replace it with*

Now**,**

**[DBS by 2]:**

**[T.S.O.B.S]:**

**General Solution:**

**Example 9: find x**

**Solution:** If

**[T.S.O.B.S]:**

**General Solution:**

**Example 10: If θ is less than 360°, solve the equation;**

**Solution: Since an exception is given in the question, It is going be restricted to values less than 360°.**

**[DBS by 3]:**

(Principal solution)

Because the General Solution is **…( Integers)**

(We have gotten this already)

(Branch Solution 1)

(Branch Solution 2)

Hence, θ ∈

(Please note, there are other values of θ less than 360°. The rest is left as assignment to the reader)

**Question 11: If find**

**Solution:** We already know that,, the next will be to know what will be

Recall that. If both sides of the equation is squared, I will obtain;

If 1 is added to both sides, you will get, that is

Is the reciprocal of

We can now say,

When

Since we’ve know the value of our Cos(y) and Sin(y), the only thing left is to substitute.

=

**Example 12: If For the value of is?**

**Solution:**

Apply cos inverse or arcCos to both sides

**Example 13: Forfind the value of** **(WASSCE Adapted)**

**Solution:**

Substituting,

By rationalization,

**Or**

Since, we can, without directly solving for y, obtain the value of and

So

When you take square root of both sides, you get:

We will take the positive result,

Because,

To avoid confusion, we are only going to be making use of the positive results. Kindly note, the negative result came as a result of the property of square roots when it is gotten from an equation.

Now,

**Example 14: find (WASSCE)**

**Solution: If**

**[Working only with the positive value],**

**Now,**

**Example 15:**  **Solve for p (UTME Adapted)**

**Solution:**

[**TSROBS]:**

**General Solution:**

**Example 16: 𝟐𝟓, Solve for k**

**Solution:** Where ever you see, call it P; **[You can choose any variable of your choice]**

Using Factorization to solve for the value of P,

Recall that it means that

**First case**:

4 d.p

**General Solution**:

**Second Case**:

**General Solution:**

Solution(s):

**Example 17: Find at least one value of x**

**Solution:** We know that

The equation will now change to,

Square both sides:

Collect like terms: Let

Using Quadratic Formula,

Recall that Cos(x) = u,

**Questions to try:**

* **Find x**
* **Find**
* **What is**
* **find y**
* **Find a**
* **Find at least 2 solutions to the equation.**

**Chapter 11**

**NUMBER BASE EQUATIONS**

A number base of a numeral system tells us about the particular symbols and notations it makes use of, in representing a value. The number base 2 tells us that there are only two unique notations 0 and 1. The most common number base is base 10, also called “DECIMAL”.

Number Base Equations or Equations involving Number Bases are those equation having number base properties on either both sides and a side of an equation.

A basic knowledge of Number base is needed here, as you know how some expressions are manipulated.

A number base can take many base “x ” , where

* Base 1 is called – Unary Scale
* Base 2 is called – Binary Scale
* Base 3 is called – Ternary or Trinary Scale
* Base 4 is called – Quarternary Scale
* Base 5 is called – Quinary or Pental Scale
* Base 6 is called – Senary/Heximal/Seximal Scale
* Base 7 is called – Septenary Scale
* Base 8 is called – Octal Scale
* Base 9 is called – Nonary Scale
* Base 10 is called – Decimal or Denary Scale

**Whenever you have an equation presented in a Number base system, you are advised to convert all systems to a system of base 10. With that, you can easily work out the equation.**

**Also, whenever you want to cancel a number or a base on both sides, you must make sure that you are canceling equivalent variables.**

**For instance:**

Since they are of same base, you conclude that

**But if the bases are unequal, then you need to convert to base 10.**

**Remember, the bases must be a Natural number.**

**i.e.,**

* **Is not negative**
* **Is not zero**
* **Fractional**

You should always have in mind, that if a number is in a particular base, none of the digits in that number must be greater than the base.

**Example 1:**  **find x**

**Solution:** Convert all to base 10

**[DBS by 2]:**  (This is a Septaquadragesimal scale)

**Example 2:** ,  **find the value of x**

**Solution:** Since the RHS is already in base 10, no need of converting it. Let’s work on making the LHS an expression in base 10.

Solving the Quadratic Equation with factorization method,

.

But because a base can never be negative, we conclude that x is just “3”

**Example 3: What is the value of P?**

**Solution: [Convert both sides to base 10]**

**[CLT]:**

and

P must be greater than 2 (Not only that, greater than 3 as well). P cannot equally be fractional nọr negative. Hence,

**Example 4:**  **solve for R**

**Solution:** The right hand side is already in base 10, you only need to convert Left hand side. Thus,

[**C.L.T] ⇒**

**[DBS by 16] ⇒**

**Example 5:**  **Solve for x if all the numbers are in base 2.**

**Solution:**

Converting everything to base 10,

Therefore**,**

But it should be in base 2. Converting

You get,

**Example 6:**  **find p**

**Solution: [C B T B 10];**

N.B: You can ignore the subscript 10 (base 10), since the standard base is 10

**[Cancel Base 10 on both sides];**

**Example 7:**  **find n. (Comprehensive Mathematics Ss1–3)**

**Solution: [C B T B 10]:**

Rewrite the equation

Because we rewrote the equation by multiplying the constant term by 3, let’s divide the constants in each factor by 3. Thus,

Because bases can never be negative nor fractional, we conclude that (Heximal Scale,)

**Example 9:**  **find P [Comprehensive Mathematics]**

**Solution:** In a situation like this, we need to convert to base 6. Before we do that, we will convert to base 10 first.

Convert to base

Therefore in base 6,

[Cancel base 6 on both sides]

Therefore,

**Example 10: find the value of q (UTME).**

**Solution:** Convert both sides to base 10;

**[CLT]:**

**Example 11: Two numbers,**  **and**  **are equal in value when converted to base 10. Find the equation connecting them. [UTME]**

**Solution:**

**Example 12: If , find x (UTME)**

**Solution:** Since x is in base 10, we say that is same as

It therefore means that in base 10,

**Example 13: Find r, if (UTME)**

**Solution:**

**Example 14: Find n, if (UTME)**

**Solution:**

**Example 15: If , find x (UTME)**

**Solution:**

**(I didn’t convert the RHS because it is already in base 10)**

**Example 16: find y (WASSCE)**

**Solution:**

**Questions to try: (Solve for x in the following)**

**•**

**•**

**•**

**•**

**•**

**•**

**Chapter 12**

**COMBINATORICS/FACTORIAL EQUATIONS**

In Mathematics, combinatorial Mathematics is a field of Mathematics that deals with the problem of selection, arrangement and operations within a discrete system. In a simple term, one can conclude that Combinatorics is all about choosing an object from a collections (Not to be mistaken for the definition of probability).

**Combinatoric Equations** are those equations having combination and permutation properties in it.

We cannot properly go into Combinatorics, without fully understanding the Concept of factorials.

**Factorial** of a number is said to be the result of multiplying a given number of consecutive integers, starting from 1, to the given number. Also, it is the product of all the positive integers, less than or equal to the number.

**Factorial Equations** are the equations having variables with factorial sign attached to it.

**Definition**:

Where n-n

Also,

. Where to stop, depends on the value of n. Let’s say n = 4, then we are to stop at (4-4)!

**Factorial of Numbers from 1 to 10**

|  |  |  |
| --- | --- | --- |
| x! | Interpretation | Value |
| 1! | 1× (1-1)! ≡ 1×0! | 1 |
| 2! | 2×1×0! | 2 |
| 3! | 3×2×1×0! | 6 |
| 4! | 4×3×2×1×0! | 24 |
| 5! | 5×4×3×2×1×0! | 120 |
| 6! | 6×5×4×3×2×1×0! | 720 |
| 7! | 7×6×5×4×3×2×1×0! | 5,040 |
| 8! | 8×7×6×5×4×3×2×1×0! | 40,320 |
| 9! | 9×8×7×6×5×4×3×2×1×0! | 362,880 |
| 10! | 10×9×8×7×6×5×4×3×2×1×0! | 3,628,800 |

Imagine what 30! Would look like!

Do you know why 0! = 1?

**Proof A**

3 letter-word “ABC” can be arranged 6 times; ABC, ACB, BAC, BCA, CAB, and CBA

2 letter-word “AB” can be arranged 2 times; AB or BA

1 letter-word “A” can be arranged once; “A”

Now, “Nothing”, how do you arrange it? There is no other arrangement other than its initial state. Which is 1.

**Proof B**

Since

Let

It means that

1×0! Is 0! And 1 is at the LHS, We conclude that 0! ≡ 1 (Take note of this)

Whenever you have x! = A, you must make sure you convert A, to a number with a factorial sign, for you to easily cancel the factorial sign.

Let’s say you are having; **x! = 6, to find x,** rewrite 6 as 3!. Because, 3×2×1 = 3! = 6.

You have, x! = 3!. The factorial signs on the RHS and LHS will cancel. Leaving x = 3.

**Example 1: If, find the value of k.**

**Solution:** Rewrite 720 as 6!

(Factorial sign will cancel factorial sign).

With that,

**Example 2: what value of x satisfies the equation?**

**Solution:**  is same as

**Example 3:**  **find a**

**Solution:**

Will cancel

But because factorial notation doesn’t work with numbers that are not whole numbers, we conclude that

**Example 4: What is the value of n?**

**Solution: If you subtract 1 from (n+2), you get (n+1). If you subtract 1 from (n+1), you get n. That is:**

**Divide both sides by “n!”**

**Expand:**

**Using Quadratic Formula,**

In this case, n cannot be negative. Hence, n= 4

**Example 5: What value of x satisfies the equation:**

**Solution:** Let x! = A;

**Using Factorization:**

**Take the positive value;**

Rewrite 6 as a number in factorial notation,

**Example 6: solve for y.**

**Solution:**

Will cancel each other;

**Using Completing of Square Method,**

**Add 4 to both sides:**

Take square root of both sides

Discarding the negative result, y = 1

**PERMUTATION & COMBINATION**

**Things to note about PERMUTATION & COMBINATION before we move on.**

**While**

From the formulae above, one can see that one COMBINATION FORMULA can be expressed in terms of PERMUTATION FORMULA. Thus,

**Example 7: Solve for n.**

**Solution:**

**Cross Multiply:**

Recall that n! = n(n-1)(n-2)! ; *[(n-2)! will cancel (n-2)!]*

**Expand:**

**By Factorization:**

(Only)

**Example 8:**  **Find n**

**Solution:**

**Cross Multiply:**

**Expand more:**

**DBS by**:

But because n must be greater than 4, n = 5

**Example 9: (WASSCE)**

**Solution:**

**Cross Multiply:**

**Using Completing of Square Method: [Add**  on both sides

**Example 10:**  **Find n** [**New Project Further Mathematics]**

**Solution:**

Will cancel

**Example 11:**  **find k.**

**Solution:**

**Example 12: find n (WASSCE)**

**Solution:**

Remember that,

Now,

⇒

**[CLT]:**

**[DAT by -1]:**

**[Using Factorization]:**

**=**

. Also,

With that, n = 8 only.

**Example 13:**  **find the value of n**

**Solution:**

Only

**Example 14: Find x**

**Solution: Remember that can be written as**

**[DAT by 3]:**

**[Using Formula Method]:**

**Remember, x must be a positive whole number. Hence, x= 2**

**Example 15: find the value of (WASSCE)**

**Solution:**

**[Cross Multiply]:**

**⇒**

**[Subtract/Cancel “r²” from both sides]:**

Now,

**You can equally say,** since

if we have ¹⁸Cr = ¹⁸C(r+2) , it will now be ¹⁸Cr  ≡ ¹⁸C(18-(r+2))

Equate r to 18-(r+2) ⇒

**Example 16: and**  **find the value of r (WASSCE)**

**Solution:**

**Divide equation I by II:**

**QUESTIONS TO TRY:**

**Chapter 13**

**Modular Equations**

**Modular Arithmetic,** sometimes referred to as modulus arithmetic or clock arithmetic, in its most elementary form, arithmetic done with a count that resets itself to zero every time a certain whole number N greater than one, known as the modulus (mod), has been reached. Examples are a digital clock in the 24-hour system, which resets itself to 0 at midnight (N = 24), and a circular protractor marked in 360 degrees (N = 360). (John L Berggren)

**Modular Equation:** In mathematics, a **modular equation** is an algebraic equation satisfied by *moduli*, in the sense of moduli problems. That is, given a number of functions on a moduli, a modular equation is an equation holding between them, or in other words an identity for moduli

(FOR DETAILED EXPLANATION, VISIT YOUR RECOMMENDED TEXTBOOK)

For you to be able to solve questions under Modular Equations, you have to bear the following in mind;

* The Sign “≡” means “Congruent”, and it is a relation between two numbers showing they give the same remainder when divided by some given number (integer).
* If It means that when a is divided by k, the remainder is b
* to find x, you must make sure that

**Example 1:**  **find x**

**Solution:**

**Example 2:, find x**

**Solution:**

**Example 3: Determine the value of x if**

**Solution:** If you divide both sides by 20, the result will not give you an integer because 10 is not purely divisible by 20. Since the Operation is “50”, you can add 50 to 10, to check if it will amount to a number that 20 can easily divide.

**[DBS by 20]:**

**Example 4 : Find x, if**  **(NECO)**

**Solution:** Since we are working with the remainder, we say,

The remainder is 0, therefore, the value of x = 0

**Example 5: find x.**

**Solution:**  8 is not purely divisible by 5, so you look for the multiples of 6 that will be added to 8, to give a number that 5 will purely divide.

8+6 = 14 (Not divisible by 5)

8+12 = 20 (Divisible by 5)

**Hence,**

**Example 6: Solve for a**

**Solution:**

But, -1 cannot be divided by 5, we look for a Multiple of 11 to be added to “-1”, to make it a number that is divisible by 5.

**Example 7: find y.**

**Solution:**

[**Minus will cancel minus];**

**Example 8: find x.**

**Solution:**

**Example 9: Find k**

**Solution:** If we divide both sides by 2, 3 will not be purely divisible by 2. So, you say,

**Example 10: Find the value of k**

**Solution:** Remember that if you havewhen a is divided by k, the remainder is b

Now, Therefore, k = 1

**Questions to try**

1. **If , find the value of x.**
2. **Find the value of p**
3. **Find k.**
4. **Find x.**

**Chapter 14**

**Modulus/Absolute Value Equations**

Modulus of a number describes how far a number is, from zero regardless of the direction.

It is the distance of a number from Zero, (How many numbers you will count after 0, to get to the number). The distance of “3” from 0 is 3units. i.e.: 1, 2, and 3.

The distance “-3” from 0 is still 3 units. i.e: -1, -2, and -3.

The Modulus or Absolute Value of any number is always positive. Even a complex number, defined as the modulus is

We can rewrite |x| to be , because the square root of any number is always positive.

The absolute value of -1 is 1. i.e. |-1| = 1

The absolute value of 1 is still 1. i.e., |1| = 1

The absolute value of -2 is 2. i.e. |-2| = 2

The Absolute value of 2 is still 2. i.e. |2| = 2

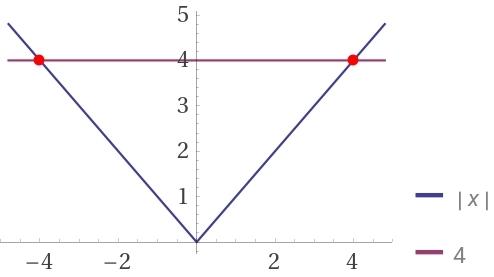
The absolute value of -3 is 3. i.e. |-3| = 3

The absolute value of 3 is still 3. i.e. |3| = 3

Etc.

Here is a look of what the absolute value graph is like;

That is the graph of When you check it, look at the dots, you will see they are at and. That is to tell you that |-4| gives 4 and |4| equally gives 4

****

**Example 1:**  **Find the value of x**

**Solution:** Absolute values are always positive, no matter the number. With that, we conclude that NO SOLUTION exists for the question above.

**Example 2:. What is the value of x?**

**Solution:** Two values will make this equation true. What web are going to do is that we are going to remove the absolute value sign and attach “±” to x. Thus;

**Case I:**

**Case II:**

**Example 3: Find x if**

**Solution:**

**Case I:**

**Case II:**

**Example 4:**  **solve for x**

**Solution:** Let’s simplify the fractions first.

When we add 1 to it, we get,

Now, to remove the absolute value sign, we say

**Case I:**

**Cross Multiply:**

**Case II:**

**Cross Multiply:**

**Example 5: find x.**

**Solution:**

**Case I:**

**Case II:**

**Example 6:**  **Find the values of y**

**Solution: Case I:**

**Cross Multiply:**

**Case II:**

**Cross Multiply:**

**Checking,**

So, -6 is a Solution.

**Checking 6,**

6 is equally a solution.

**Example 7: Find the value of x**

**Solution:** (for case I), and (for case II)

But when we look at case 2, we are going to notice that the outcome doesn’t make any sense, because x-x is 0.

Now,

**Example 8: If what is the value of x**

**Solution:**

**Solution:**

**Checking,** but Hence, 3 is the only solution.

**Example 9:**  **find x**

**Solution: We are going to have 4 Cases here, since the absolute value is 2.**

**Case I:**

**Case II:**

**Case III:**

**Case IV:**

Let’s check the values we got from I & II

(Not a solution)

With that, we conclude that the only value of x is 5

**Example 10: What is p, if**

**Solution:** Kindly note that you can’t just start eliminating the absolute value sign, without making sure that every number attached to it ( multiplying or dividing it ), is removed.

**Divide both sides by**

**Case I:**

**Case II:**

**Let’s verify,**

**In case II,**

So, 2 and -10 are both correct.

**Example 11:**  **find a**

**Solution:** We have 2 expressions with absolute value sign, so our cases are going to be 4.

Case of + and +, Case of + and –, Case of – and –. and lastly, Case of – and +

**Case I:**

**Case II:**

**Case III:**

**Case IV:**

With that, the values of. (Verify it on your own)

**Example 12: Solve for x**

**Solution:**

**Case I (+ and +):**

**Case II (+ and -):**

**Case III (- and -):**

**Case IV (- and +):**

**Example 13: If find the value of x**

**Solution: Case I:**

**Case II:**

Using Quadratic Formula,

**Example 14: Solve for x.**

**Solution:** Zero has a neutral property. It is neither negative nor positive.

The solution to the equation above is just to solve, without having different cases.

**Example 15: What is the value of u?**

**Solution:**

**Case I:**

**Case II:**

to 2 d.p

**Questions to try:**

**• |x+4| = |5|, find x.**

**• |5x – 3| = |2x|, find 6x.**

**• 2|y – 7| = 10, show that y is an integer.**

**• find k.**

**• What is the value of x?**

**Chapter 15**

**Polynomial Equations**

A Polynomial is an expression having more than two algebraic terms, in most cases, the sum of several terms that contain different powers of the same parameter/variable. It is an expression made of different Constants, variables and exponents (that are not negative nor fractional) joined together using different mathematical symbols like, +, –, × and ÷.

In a simpler term, one can tag it the combination of different Monomials, Binomials and Trinomials to work as a single function.

|  |  |
| --- | --- |
| Monomial |  |
| Binomial |  |
| Trinomial |  |
| Polynomial |  |

Once the number of terms in the expression exceeds 4, then it is a “POLYNOMIAL EXPRESSION”

Note, a Quadratic expression is still a Polynomial. Cubic, Quartic, Quintic and all other higher degrees are part of Polynomial. Just that anyone that has mixed terms, especially with different powers, we call it a “Polynomial”.

Is a Polynomial of first degree

Is a Polynomial of second degree

Is a Polynomial of third degree

Is a Polynomial of fourth degree

**Etc.**

You name them following their dominating (First) term.

On that note,

If

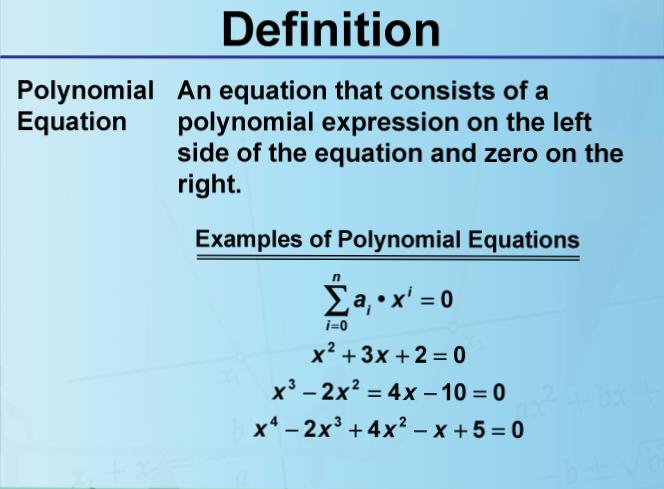
Wherewe conclude that the function is a Polynomial of nth degree

Also, is the leading coefficient?

Is the dominating term

Is the constant term and the y-intercept of the graph

**Polynomial Equations,** is therefore the kind, having Polynomial expressions at the RHS, and “0” at the LHS. Vice Versa, or having expressions at both sides, with a sign of equality as a conjunction. The equation of a polynomial is basically just the polynomial written in the form:

Where n is the degree of the polynomial, are the coefficients of the terms, and is the constant term. In this form, the equation represents a relationship between the variable x and the coefficients of the terms. When you solve a polynomial equation, you’re finding the values of x that make the equation true.

There are methods one can adopt, in solving Polynomial equations.

We have **Analytical, Numerical & Graphical.** Since all we are doing is under the scope of SSCE, We can only use Analytical & Graphical. Though we will only be making use of Analytical Method. (See page 2)

We have a known method used in Solving Quadratic/Linear Equations. Whenever we are faced with equations having a variable with a power that is above 2, we need to reduce it to an equation that we know. Through what? Through Factorization.

It is true that not every Polynomial can be factorized, when considering coefficients that are Integers. We should equally note that such questions do not come out in exams, if it is within the Scope of SSCE.

One of the ways you use in factorizing a Polynomial is through “GUESS & CHECK” or “TRIAL & ERROR (Improvement)”.

If you have:

You can easily solve it without T&E because you can factorize it with ease.

With that,

It has 3 roots, showing it is a Cubic equation. What if the terms there are much that we can’t factorize by mere looking at it? Let’s find out.

If you have:

Here, you have to look for the possible factors of 6 and substitute into the equation. If any of them gives you Zero, then know that it is one of the roots of the equation. In most cases, WAEC/NECO will provide one root, then we solve for the rest.

Here is the Trial and Improvement;

Let’s start with “+1”;

and

It shows that “+1” is not a Solution to the equation.

Let’s check “-1”;

It means that “-1” is a Solution to the equation

Let’s check “2”:

“+2” is not a Solution.

Let’s check “-2”;

Now, it should have 3 roots. You are to solve for the other root by testing, ±3, ±6.

What if you don’t have the time to test those values, what are you going to do? Let’s us find out.

We know thatis a linear expression and is a linear equation.

is a quadratic expression, whileis a quadratic equation.

**Do not forget that ax²+bx+c = 0, is also, but the standard form of a quadratic equation.**

When you have a Polynomial of 3rd degree (Cubic), and you know a root of that equation, you can generate a factor of the Polynomial from the root. If you have a function, and then 1 is a solution, we say that . Hence, is a factor.

If 8 is a root of, then is a factor of

If -10 is a root of, then is a factor of

If 28 is a root of then is a factor of

So, is a root of it means that “is a factor of

**“It is a factor” means it can divide the polynomial without leaving a remainder.**

(**STUDY POLYNOMIALS FOR FURTHER EXPLANATION)**

Since we have established that is a factor of

Our question should be “which other factors are we going to multiply with to give us .

It, of course, will not be a linear factor/expression, because linear × linear is Quadratic.

But linear × Quadratic is Cubic.

Now, we are going to multiply with a certain quadratic expression in order to obtain.

We say,

**Expand:**

**Arrange:**

**Equate terms with similar coefficients:** … equation I

Equation II

Equation III

Equation IV

From equation I, So in equation II,

We can use equation III to solve for “c”, but since we’ve already seen that c=6, no need of solving further.

With that,

Remember that the standard form of a quadratic equation is

You will have this,

**Solving for x,**

In conclusion, the values of x in the equation, are

**Example 1: If is among the roots of the equation Find other values of x**

**Solution:** Since is a root, it means that and is a factor of

The next step is to find another expression, such that when multiplied with gives

Remember that **linear expression × Quadratic expression will give a Cubic expression.**

**Equate terms with similar coefficients:**

From I, . In substitution to II,

From IV,

We have gotten our we can now go on to generate the quadratic equation and proceed with the solution.

Using quadratic formula, if then,

With that, values of x, aside -10 are

**Example 2: If is a factor of the polynomial Find other roots of the equation apart from -6**

**Solution:** It is true that solving this particular question using the idea we earlier used will make it look as if it is no more the usual “Quadratic equation”, but I want to make things clearer.

Linear × Linear is Quadratic, so will look for an expression, such that when it is multiplied with gives.

**Equate terms with similar coefficients:**

From I, in substitution into II,

Since we now know and we will now have,

So, 12 is another root of the equation aside

**Example 3: is exactly divisible by find the constants p and q, and find other 3rd root of x. (WASSCE)**

**Solution:**

**Expand:**

**Equate terms with similar coefficients:**

In equation III, , and in equation I,

So,

a = 1 and b = -4. Recall that and

We say, and

If and the cubic equation will now be;

Remember that ,it means that

Is a factor of

Also, and

So I have

Hence, 4 is another root of the equation

**Example 4: If the equation has equal roots, find the value of p (WASSCE)**

**Solution:** For a Quadratic equation to have 2 equal roots, it means that the Quadratic expression at either RHS or LHS is a perfect square. Now, for a Quadratic expression to be a perfect square, it means that its discriminant must be equal to zero.

The discriminant of a quadratic expression is,

Now,

**CLT:**

**Divide all through by “4”:**

**Solve by factorization,**

and

**Example 5: If and are 2 of the factors of the expression**

**Find the values of k and y**

**Solution:** Linear × Linear × Quadratic = Quartic. We therefore have it that,

**Expand & Equate:**

**Factorize & Equate:**

**Equate the terms with similar coefficients;**

(Keep note of this)

(Also note this)

From equation IV,

Now, from equation II, Therefore

From equation III, Therefore

Hence, the values of k and y are: respectively

You can equally have a Polynomial whose leading coefficient is not equal to 1. (Not equal to zero either).

**Example 6:**

**If is a factor of the polynomial find other roots of the equation apart -6 (WASSCE)**

**Solution:** Since we have been given the linear factor that will be multiplied to a quadratic factor to give us the polynomial , we say:

**Equate the terms with similar coefficients:**

From I & IV, we have already gotten, but we need “b”. From either equation II or III, but we will choose II, we have

If the quadratic expression will now be While the equation is

Let’s use **completing the Square Method:**

Add to both sides ⇒

**Example 7: If are factors of find the values of L and k (UTME)**

**Solution:**  If and are factors it means that they can divide the quadratic expression without a remainder. Meaning that the remainder will be 0. Also, to check whether they are truly factors or not, we will say,

and substitute them into the expression and equate it to zero.

**First case**  ⇒

**Second case** ⇒

**From the first case,** (When you DAT by 4)

**In substitution into equation II (second case**):

**To find we substitute into any of the equations:**

With that,

**Example 9: If 3 is a root of the equation Find other root and value of p**

**Solution:** If 3 is a root of the equation, it means, and when 3 is substituted into the equation, it balances. Therefore,

We now have,

and

Conclusion: The other value of x apart from 3 is 14 and the value of p is 17.

**You can also approach the question this way:**

Since the factor of the given polynomial is you can look for a factor that will be multiplied with in order to give

**Equate the terms with similar coefficients:**

If a = 1 and b = -14, will now be

Remember that so

For the other root,

**Example 10: The equation has equal roots, find the values of a.**

**Solution:** If the equation is having equal roots, it means that the expression at the Left hand side is a perfect square.

Now, for an expression (Quadratic) to be a perfect square,

That is:

Here,

**Example 11: If** **is a factor of the polynomial Find other roots of the equation (500 Boosters Problem Solving in Mathematics by Nnamdi Onyia)**

**Solution:**

**Equate the terms with similar coefficients:**

From I, a= 1. From IV, c = -6. Now, what is b?

From II,

The quadratic expression will now be,

**Solving for x,**

So, the other values of x aside 1 are 3 and -2

**Example 13: What are K and L respectively if (UTME).**

**Solution:**

Re arrange ⇒

Equate the terms with similar coefficients ⇒

Hence, K and L are -12 and 9/2 respectively.

Note: I didn’t pick 8x² = 8x² because both of them will cancel each other.

**Example 14: Three consecutive positive integers k, l and m are such that . Find the value of m**

**Solution:** Let the numbers be x, (x+1) and (x+2)

Remember that the question said “positive integers”, we will only be taking x= 5

Now, the numbers will be

Where 5 = k, 6 = l and 7 = m.

Therefore, m = 7.

**Example 15: If. (UTME)**

**Solution:** a² – b² will be

Conclusion: The expression evaluates to or

**Example 16: If has x= -2 as a solution, then the equation has**

1. **x = -4 as a solution also.**
2. **3 roots all different.**
3. **3 roots with two equal and the third different.**
4. **3 roots all equal (UTME)**

**Solution:** If x= -2, it means that x+2 = 0 and (x+2) is a factor of (x³–12x–16).

(x+2) will be multiplied with a quadratic expression to give you a Cubic expression, which is

Since the RHS is not having any term with x², you equate to 0

Equate terms with similar coefficients:

Remember we are looking for

You can either use II or III. From II,

The quadratic equation becomes

Solving for x,

Remember that already, -2 is a solution to the equation. So, the answer becomes option **(C)**

**(3 roots with 2 equal roots (x= -2 and -2) and the 3rd root (x = 4) different)**

**Example 17: If and are factors of the expression , find the sum of p and q (UTME)**

**Solution:** Take a look at this, if is a factor of x² –1, what does it imply?

It means that (x+1) can easily go into x²–1 without leaving a remainder. Also, if you say

And when you substitute (-1) into x²–1, the result will be 0. How?

(-1)² –1 ⇒ 1–1 = 0.

If you is a factor of x³+px²+qx+1, it means that

Also, when x = 2 is substituted, the polynomial x³+px²+qx+1 will be equal to zero.

We have

From II, In substitution,

Remember that p = q. Therefore,

The sum of p and q =

**Questions to try:**

**• Find other roots of x³ –1 = 0, aside x = 1.**

**• , Solve for a and b**

**• If x = 1, x = -1 and x = 2 are the roots of the equation ax³+bx²+cx–1 = 0, find a,b and c ( UTME Adapted)**

**• If x² + 16 = 0, find the value of x**

**• Find the value of m**

**Chapter 16**

**Matrix Equations**

According to Oxford dictionary, a matrix is a rectangular array of quantities or expressions in rows and columns that is treated as a single entity and manipulated according to particular rules. In short, it is the rectangular arrangement of numbers in a fixed into a fixed number of rows and columns.

When the numbers are arranged vertically, it forms the columns while horizontal arrangement forms the rows.

It is good to note that matrices are named where m is the number of rows and n is the number of Columns.

In a situation where the elements that the row is having is less than 2, it is called **Column** **Matrix,** when the Column is having numbers that are not up to 2 in number, the matrix is said to be a **Row Matrix.**

**Examples:**

Is a 2-by-2 matrix. Is a Column matrix because it is (It is also a Vector)

Is a row matrix because it is 1-by-5 (It is also a Vector) is a 3- by-2 matrix

Is a 3-by-5 matrix. Is a 2-by-3

**Matrix Equations** are therefore equations involving matrix properties.

Under this very chapter, we are going to treat the equation in 3 forms:

* Equations involving addition & subtraction
* Equation in determinant form
* Equation in the form of

**Addition & Subtraction**

**Example 1: Find the value of k**

**Solution:**

**Example 2:**

**Find the values of a and v**

**Solution:**

**Example 3: Find the value of x and y**

**Solution:** If we look at the two matrices,

With that, the sum of x and y will be,

**Example 4: The matrix, when added to gives Find the product of**

**Solution:**

Equating the elements,

and

**Example 5: Suppose Find the value of 2x**

**Solution:** , if we add the 2 equations we get:

Therefore,

**Equations in Determinant form**

Suppose you have a matrix determinant is

The determinant of the matrix is

The determinant of the matrix is

But that is not why we are here!

Whenever you are asked a question involving determinant of matrices, it can come in 3 popular forms.

1. If the matrix is K and K = , then you will be asked to find
2. If the matrix is K and K =, you may be asked to find |k|. the vertical bars means “determinant”
3. They can equally go straight and ask you to find

* Whichever way you see it, just know it is talking about determinant.

Note: You can only compute the determinant of a square matrix. I.e., 2-by-2, 3-by-3, 4-by-4, etc.

**Example 6: If** **find the value of m**

**Solution:**

**Example 7: When**  **find the value of u**

**Solution:**

**Example 8: find the value of a**

**Solution:**

Using quadratic formula,

The positive value of a is 2

**Example 9:**

**If P = Find x if**

**(500 Boosters Problem Solving in Mathematics by Nnamdi Onyia)**

**Solution:**

**Example 10: If , find the value of x. (500 Boosters Problem Solving in Mathematics by Nnamdi Onyia)**

**Solution:**

**CLT:**

**Example 11: A = if is 7. Find the value of y**

**Solution:**

**Example 12: If the determinant of the matrix**  **is equal to 13, find the value of x.**

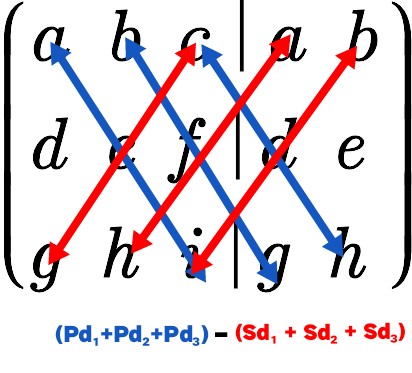
**Solution:**

**CLT:**

**Equations in Determinant form ( 3 – by – 3)**

The Determinant of the matrix is =

Pierre Sarrus, a French Mathematician gave a more comprehensive method which states that, if you have a matrix

The determinant is:

When you have a matrix in that form, what you are to do is write out first 2 columns of the matrix and draw your diagonals.

In the diagonals above, 3 diagonals are principal while 3 are Secondary.

From what we know as the determinant of 2-by-2 matrices; “Pd.–S.d”, here,

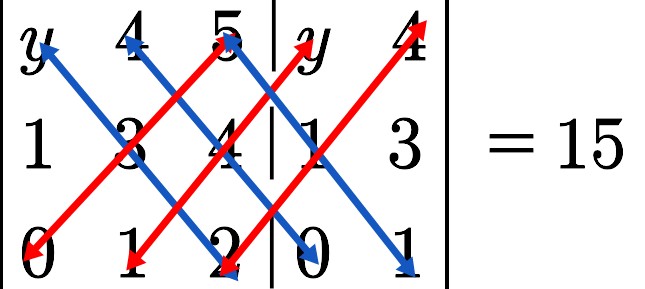
We are having more than one diagonals, so it will be, “The sum of the principal diagonals” – “The sum of the Secondary diagonals”.

Thus,

**Example 13:**

**Let A** **if the determinant of A is 15, find the value of y.**

**Solution:**

****

**Example 14: Given that**

**Find the values of the constant “k” (WASSCE)**

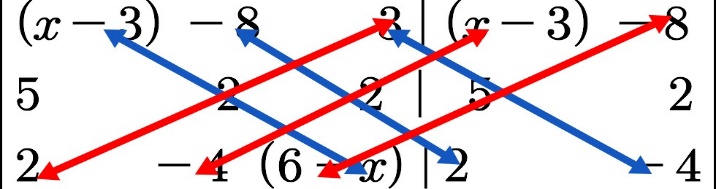
**Solution:** Using the conventional method of evaluating 3 by 3 matrices, we have:

Using “ac” method of factorization to solve for k,

Because I multiplied “c” with “a”, I will divide every constant in the linear factors by “5”,

**Example 15: If**  **find the value of x (WASSCE Adapted)**

**Solution:** Using Sarrus’ rule, let’s simplify the determinant first.

****

**Equate it to -44:**

**Equations in the form of (VECTOR)**

**(2 – by – 2) and (3 – by – 3)**

In this very part, we will solve questions almost like simultaneous equations. We shall be using the idea of “Inverse of a matrix”.

Kindly note, this chapter doesn’t in anyway intend to solve simultaneous equations. It is strictly for a simultaneous equations in **Matrix** form.

From what we already have as fact, if then

Where means inverse function of the matrix A and b is a Vector

The Inverse of the Matrix, M =

Where is the determinant of the matrix

Whenever you have a question like

To find

and

**Example 16: When , find the values of x and y.**

**Solution:**

**Example 17: If find the sum of**

**Solution: ⇒**

The sum of a and b is 7

**Example 18: When Show that k is an integer**

**Solution:**

and

Also,

So, k =4 in both cases.

**Example 19:**

**Given that find the value of m and n**

**Solution:**

**Example 20: If find the values of x,y, and z.**

**Solution:**

Remember that the main idea of this chapter is to teach you “Matrix Equations”, and not Matrices and its Operations. You must learn the basics of what matrices entails, in order for you to have a proper understanding of what the equations is all about. So, read (study) up “Inverse” of 3-by-3 Matrices so you would understand what you will encounter in less than 5 seconds.

The Inverse of is

**Example 21: Given that find the values of a,b and c**

**Solution:**

The Inverse of is , you have,

**Example 22:**

**When , find the sum of and**

**Solution:**

The Inverse of is

So,

Therefore, and

**Questions to try**

**• Find**

**• solve for m and n.**

**• . What is the value of x?**

**• Show that x is irrational if.**

**• Find x and y if**

**Chapter 17**

**Other Equations**

In this very chapter, we are going to be dealing with some equations that are not within the scope of SSCE, but the Author decided to include it because of;

* Its technicality
* The fact that the readers may meet in their POST UTME or other Exams.

**Example 1: If, find the values of x**

**Solution:** Introduce logarithm to both sides ⇒

From there, and

and

So, in both cases, x = 1. But there is a very interesting thing about the equation.

Which makes -1 a valid solution, but how do we justify that? Let’s find out.

Multiply both powers by

From I, From II, It means that

Now, cancel x at the bases ⇒

**Cross Multiply**:

Hence,

**Example 2: Consider find an approximate value of x (Real answer only)**

**Solution:**

Multiply both powers by 3:

Equate the power/base: 5 d.p

**Example 3: If find the value of k**

**Solution:**

We all know that

Where x is any number (that is not Infinity)

We will now have:

⇒

Multiply all through by 2:

**Example 4: Given that find the value of x.**

**Solution:** Rearrange the equation by making subject of formula:

Take square root of both sides,

Discard the negative result,

If you substitute x into the expression at the RHS, you have:

If you continuously substitute x, you will get,

For that,

Solving for x,

But -2 is extraneous. Hence,

**Example 5: When find the value of a**

**Solution:**  This is Tetration/repeated exponents

Since

It means that

Taking the positive value,

**Example 6: If** **find the value of p**

**Solution:**

Square both sides:

But you have:

**Example 7: If solve for x.**

**Solution:** If you look at the powers of 3, you will notice it is having something in common. (They are terms in a Geometric progression and it tends to Infinity).

Consider

We now have:

Equate the powers:

But 0 is extraneous. Hence, x = 5

**Example 8: Consider, find the value of x.**

**Solution:** Let x = U+V. If so,

Equating it with the main equation we have:

Equate the terms with similar coefficients:

From equation I,

From equation II,

Solving for

But because and are conjugates, you have it that and

Remember we are looking for U+V?

**Example 9: Find Sinx – Cosx, and express in terms of pi**

**Solution:**

Also,

In conclusion,

Note, it is having 4 values:

**Example 10: Find the value of Cos(x)**

**Solution: Square both sides**,

Shift the ones that are not radical to the other side:

**Square both sides again**,

**Expand**:

CLT:

Using Quadratic Formula,

But is extraneous. From Trig. Identity

**Example 11:**

**Solve (MTH 101, IMS Dept. Publications ESUT)**

**Solution:**

**Example 12: find**

**Solution:**

Square both sides:

But

**Example 13: Find the value of x to 3 decimal places**

**Solution:** Let

Multiply both sides by

But

Introduce natural logarithm on both sides ⇒

And

Conclusion:

**Example 14: If find**

**Solution:**

**Example 15: Find x and y**

**Solution:** There is something Interesting about this particular equation. It is different from what we’ve been treating/dealing with, as the vector that is being multiplied with a square matrix, is usually where the missing variables are located. But this time around, a missing variable is in a square matrix. Also, a number is in a vector. Let’s check it out.

The inverse of

From II,

Substitution, into I,

Hence, x= 3 and y = 1

Kindly note, if the context were to be “Simultaneous equations”, then going this way isn’t advised because it takes a lot of time. Handling it like a normal Simultaneous equation would have been better, but we needed to go this way because we deem it fit letting the reader know that Matrix equations can come in so many forms.

**Keep calm and love Mathematics!!**

**Maths Atọka**

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